Australian Government

Geoscience Australia

Geodetic VLBI, Earth rotation and Sagnac effect

Oleg Titov (Geoscience Australia)

Hana Krásná (Technische Universität Wien, Austria
Astronomical Institute of CAS, Czech Republic)

Astronomical Institute of the Czech Academy of Sciences

TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

Introduction

1. Geodetic VLBI outputs two observable values: group delay and delay rate (frequency shift) since 1979
2. Only group delay is used for geodetic VLBI data analysis
3. The majority of VLBI data analysts ignore the delay rate without any excusable reason

$$\phi(\omega, t) = \phi_0(\omega_0, t_0) + \frac{\partial \phi}{\partial \omega}(\omega - \omega_0) + \frac{\partial \phi}{\partial t}(t - t_0) \ ,$$

16 channels (S/X)

$$\tau_{\rm pd} = \phi_0/\omega, \qquad \tau_{\rm gd} = \frac{\partial \phi}{\partial \omega}, \qquad \dot{\tau}_{\rm pd} = \frac{\partial \phi}{\partial t} \ .$$

3 parameters to be estimated

Phase **Group delay** **Phase rate (delay rate, frequency shift)**

$$\Omega = 7.292 \cdot 10^{-5}$$

Introduction

We think this is a terrible mistake…

$\Omega = 7.292 \cdot 10^{-5}$

Group delay modelling (referred to t_1)

$$\phi(\omega, t) = \phi_0(\omega_0, t_0) + \frac{\partial \phi}{\partial \omega}(\omega - \omega_0) + \frac{\partial \phi}{\partial t}(t - t_0) \ ,$$

$$\tau_{21} = \frac{-\frac{(\mathbf{b}(t_1)\cdot\mathbf{s})}{c}\left(1 - \frac{2GM}{c^2 r} - \frac{\langle {\mathbf{V}_\oplus}^2 \rangle}{2c^2} - \frac{(\mathbf{V}_\oplus \cdot \mathbf{w}_2)}{c^2}\right) - \frac{1}{c^2}(\mathbf{b}(t_1)\cdot\mathbf{V}_\oplus)\left(1 + \frac{(\mathbf{V}_\oplus \cdot \mathbf{s})}{2c}\right)}{1 + \frac{1}{c}(\mathbf{s}\cdot(\mathbf{V}_\oplus + \mathbf{w}_2(t_1)))}$$

IERS Conventions 2010

Closure delay (used to detect source structure)

$$\Delta\tau = \tau_{21} + \tau_{32} - \tau_{31} \approx -\frac{(b_{21} \cdot s)}{c}\frac{((w_3 - w_2)\cdot s)}{c} = -\frac{(b_{21} \cdot s)}{c}\frac{(\dot{b}_{32} \cdot s)}{c}$$

Not equal to zero! (group delay × delay rate)

$$\Omega = 7.292 \cdot 10^{-5}$$

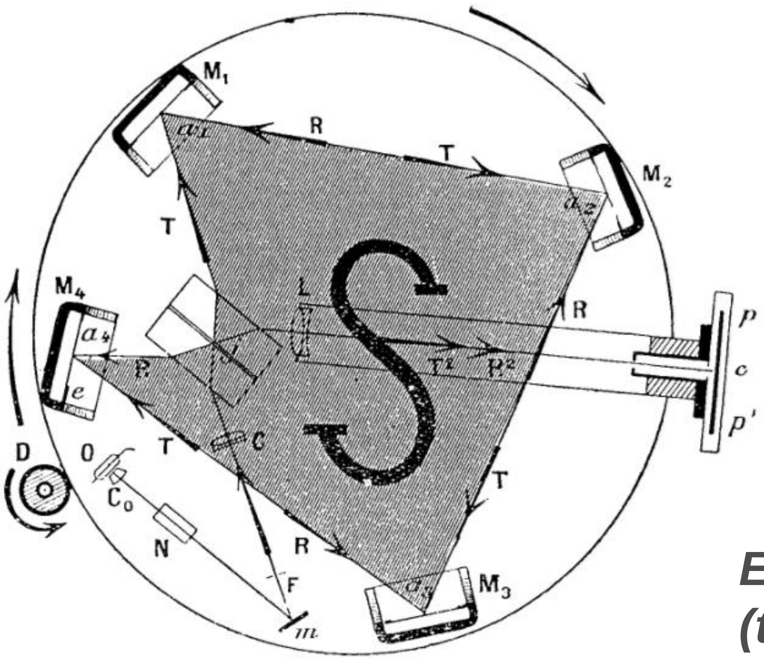
Sagnac effect

George Sagnac (1913)

Sur la preuve de la réalité de l’éther lumineux par l’expérience de l’interférographe tournant *Comptes Rendus*, **157**: 1410–1413

On the proof of the reality of the luminiferous aether by the experiment with a rotating interferometer

Note by G. SAGNAC, presented by E. BOUTY.



$t_2 = t_1$ Fixed platform

$t_2 \neq t_1$ Rotating platform

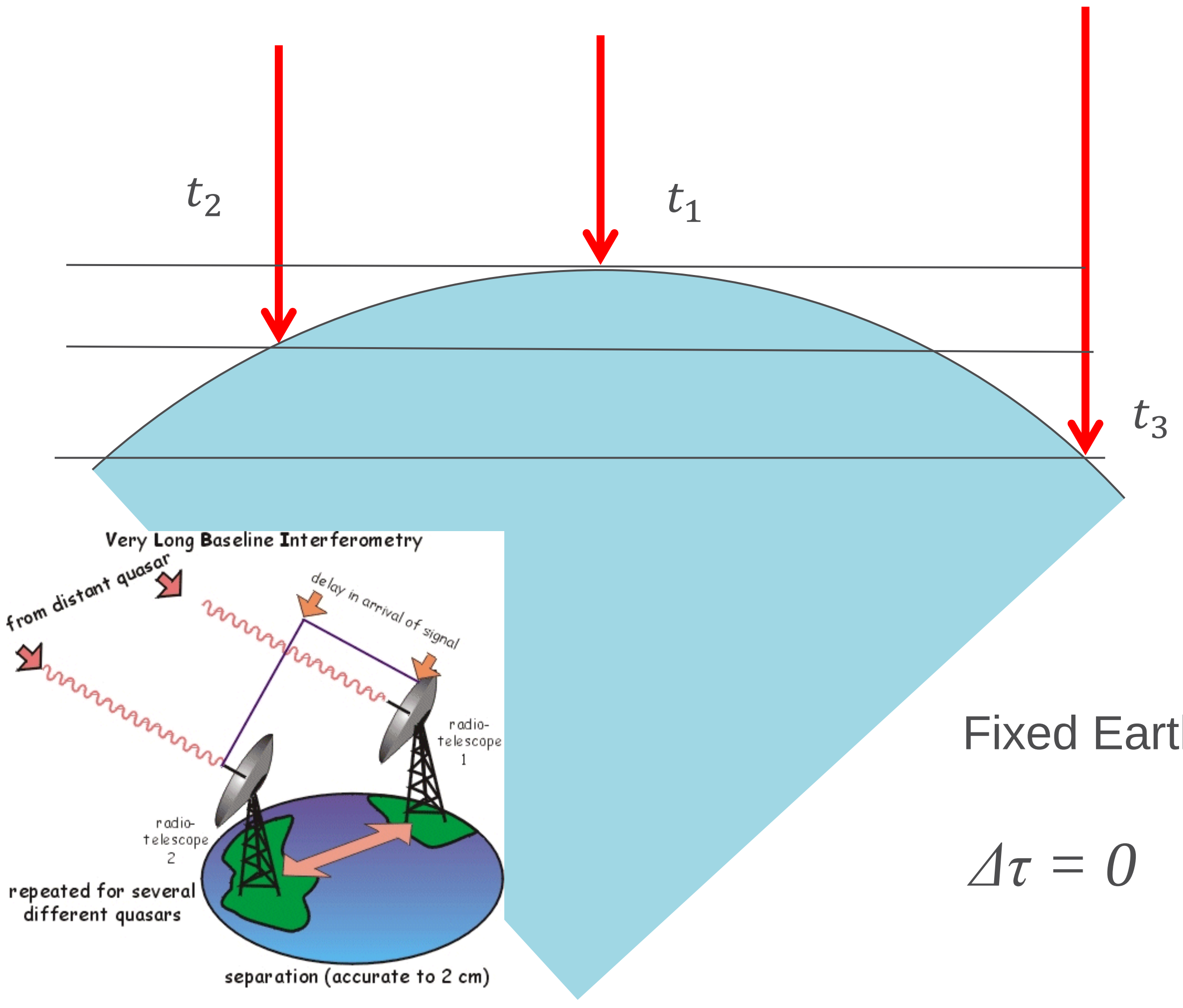
^ Langevin, Paul (1921). "Sur la théorie de la relativité et l'expérience de M. Sagnac". *Comptes Rendus*. **173**: 831–834.

^ Langevin, Paul (1937). "Sur l'expérience de M. Sagnac". *Comptes Rendus*. **205**: 304–306.

Explanation in terms of special relativity (the concept of rotating reference frame)

$$\Omega = 7.292 \cdot 10^{-5}$$

Closure delay and the Sagnac effect



$$\tau_{21} = t_2 - t_1$$

$$\tau_{31} = t_3 - t_1$$

$$\tau_{32} = t_2 - t_1$$

$$\Delta\tau = \tau_{21} + \tau_{32} - \tau_{31}$$

Fixed Earth	Rotating Earth
$\Delta\tau = 0$	$\Delta\tau \neq 0$

$$\Omega = 7.292 \cdot 10^{-5}$$

Group delay and the Sagnac effect

$$\Delta\tau = \tau_{21} + \tau_{32} - \tau_{31} = -\frac{(b_{21} \cdot s)}{c}\frac{((\Omega \times b_{32}) \cdot s)}{c}$$

$$\tau_{21} = -\frac{(b_{21} \cdot s)}{c}\frac{((\Omega \times r_2) \cdot s)}{c}$$

Ω is the instantaneous vector of the Earth rotation

The non-zero group delay is the manifestation of the Sagnac effect! (now without the closing path)!

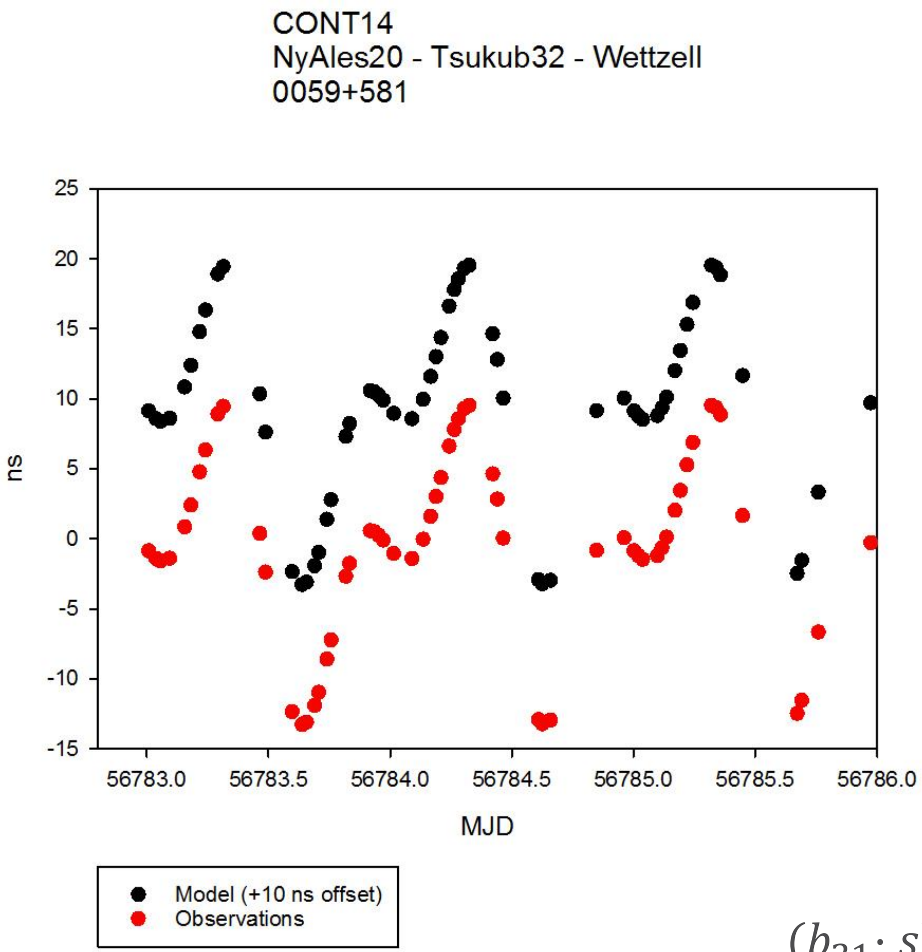
$\boldsymbol{\Delta\tau \approx 10^{-8}\ sec}$ $\boldsymbol{\sigma_{\Delta\tau} \approx 10^{-12}\ sec}$

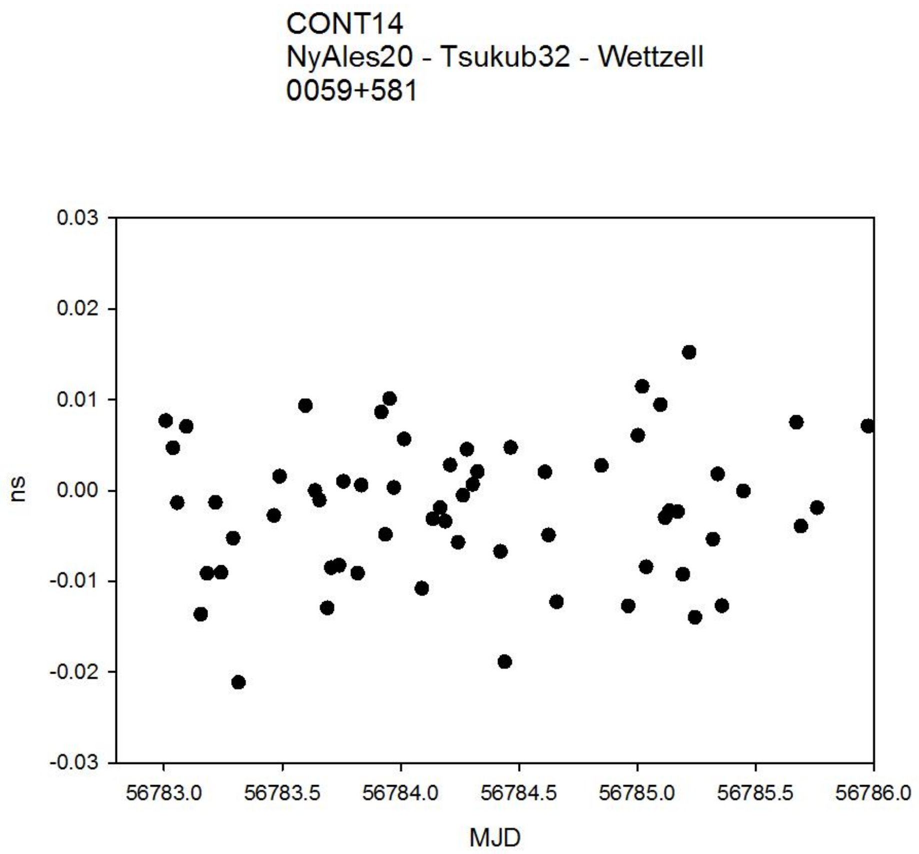
$\boldsymbol{\sigma_\Omega = \frac{\sigma_{\Delta\tau}}{\Delta\tau} \approx 10^{-4}\Omega \approx 10^{-8}}$ **Precision is moderate**

$\boldsymbol{\Omega = 7.292 \cdot 10^{-5}}$

Closure delay and the Sagnac effect

VieVS & OCCAM





$$\Delta\tau == -\frac{(b_{21} \cdot s)}{c} \frac{((\Omega \times b_{32}) \cdot s)}{c}$$

$$\Omega = 7.292 \cdot 10^{-5}$$

Delay rate and the Sagnac effect

$$\phi(\omega,t) = \phi_0(\omega_0,t_0) + \frac{\partial\phi}{\partial\omega}(\omega-\omega_0) + \frac{\partial\phi}{\partial t}(t-t_0)\ ,$$

$$\frac{\Delta f_{21}}{f} = -\frac{((\mathbf{w}_2-\mathbf{w}_1)\cdot\mathbf{s})}{c}\left(1-\frac{2GM}{c^2 r}+\frac{(\mathbf{V}_\oplus\cdot\mathbf{s})^2}{c^2}-\frac{\langle\mathbf{V}_\oplus{}^2\rangle}{2c^2}-\frac{(\mathbf{V}_\oplus\cdot\mathbf{s})}{c}-\frac{(\mathbf{w}_2\cdot\mathbf{s})}{c}-\frac{(\mathbf{b}_{21}\cdot\mathbf{s})}{c}\frac{(\mathbf{a}_\oplus\cdot\mathbf{s})}{c}\right)+$$

$$+\frac{(\mathbf{b}_{21}\cdot\mathbf{s})}{c}\frac{(\mathbf{a}_\oplus\cdot\mathbf{s})}{c}+\frac{(\mathbf{b}_{21}\cdot\mathbf{s})}{c}\frac{(\mathbf{a}_2\cdot\mathbf{s})}{c}-\frac{(\mathbf{b}_{21}\cdot\mathbf{a}_\oplus)}{c^2}-\frac{((\mathbf{w}_2-\mathbf{w}_1)\cdot\mathbf{V}_\oplus)}{c^2}-$$

$$+\frac{((\mathbf{w}_2-\mathbf{w}_1)\cdot\mathbf{V}_\oplus)(\mathbf{V}_\oplus\cdot\mathbf{s})}{2c^3}-\frac{((\mathbf{w}_2-\mathbf{w}_1)\cdot\mathbf{s})(\mathbf{V}_\oplus\cdot\mathbf{s})^2}{2c^3}+f_{grav}$$

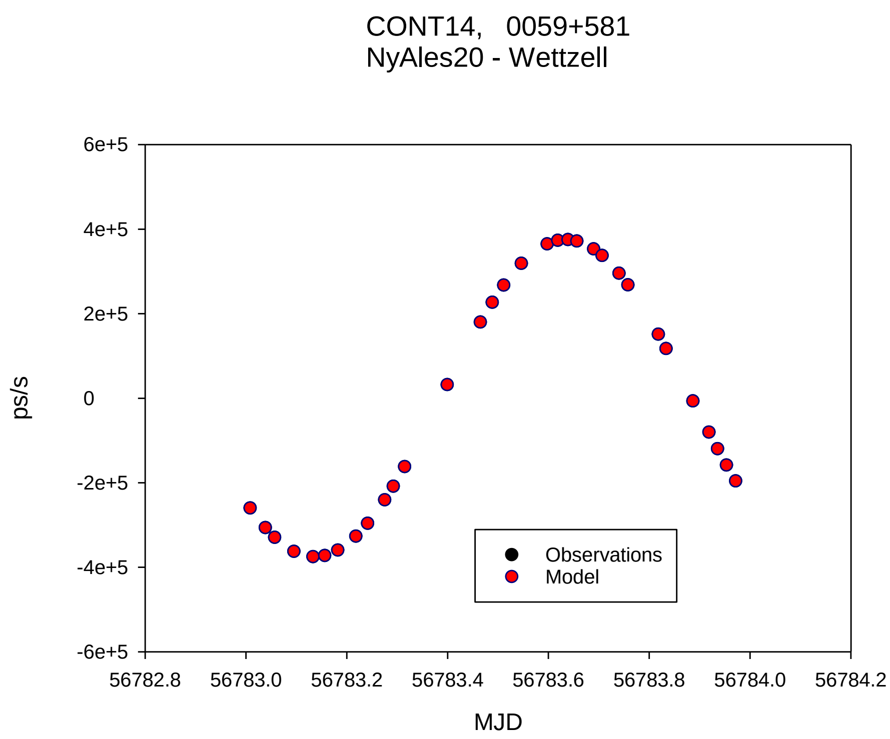
Major term now is the Sagnac effect!

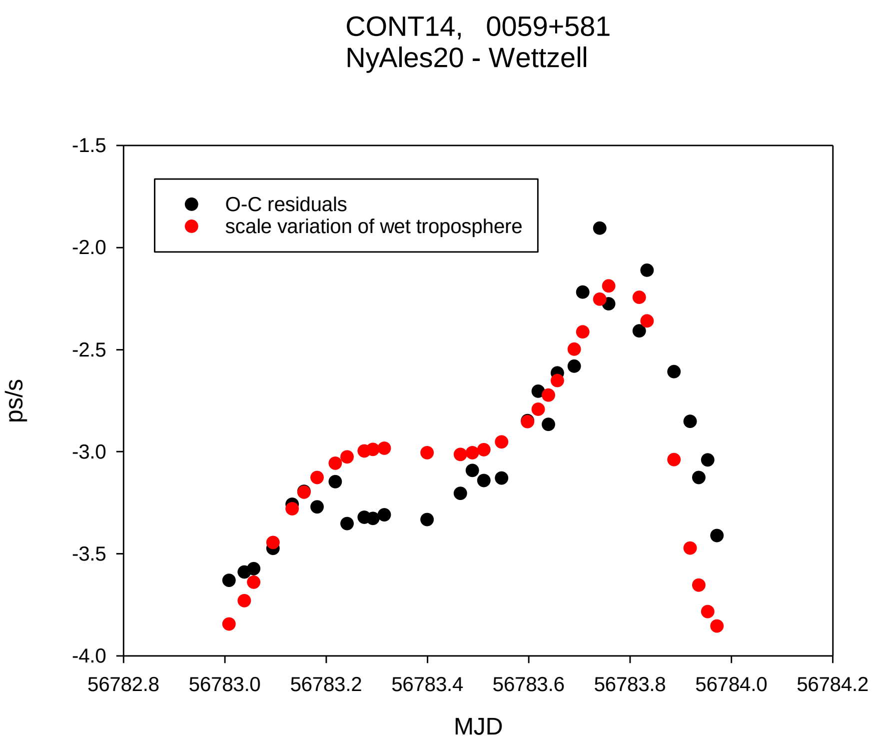
$$\frac{\Omega \mathrm{x}(r_2-r_1)}{c}\sim 10^{-6} \qquad \frac{\Delta f}{f}\sim 10^{-14}\div 10^{-15} \qquad \sigma_\Omega = \frac{\sigma_{\Delta f}}{\Delta f}\approx(10^{-8}\div 10^{-9})\Omega$$

One single delay of 20 minutes. Precision is comparable with the ring laser

$$\Omega = 7.292\cdot 10^{-5}$$

Relativistic model for delay rate





Residuals of 1-5 ps/s origin by the troposphere
Advanced mapping function should be developed

$$\Omega = 7.292 \cdot 10^{-5}$$

Sagnac effect and absolute rotation

$$\frac{\Delta f}{f} \sim 10^{-14} \div 10^{-15} \qquad \frac{w_2 - w_1}{c} = \frac{\Omega \times b_{21}}{c} \sim 10^{-6} \qquad \frac{b_{21}}{c} \sim 10^{-2}$$

To be detected Ω should have a magnitude of $\sim \mathbf{10^{-13}}$

Potentially detectable rotation effects:

- Geodetic precession $\Omega \approx 10^{-14}$ Hz
- Frame dragging $\Omega \approx 4 \cdot 10^{-15}$ Hz
- Rotation of the Galaxy $\Omega \approx 10^{-15}$ Hz

$\mathbf{\Omega = 7.292 \cdot 10^{-5}}$

Radio source catalogues

$$\tau \approx \frac{bs}{c} \sim 10^{-11} \div 10^{-12} ps \qquad \sigma_s \propto 10^{-9} \div 10^{-10} \text{ rad}$$

$$\frac{\Delta f_{21}}{f} \approx \frac{(w_2 - w_1)s}{c} \sim 10^{-14} \div 10^{-15} \qquad \sigma_s \propto 10^{-8} \div 10^{-9} \text{ rad}$$

What is the benefit?

1. Free of all phase-related troubles (clock, troposphere, phase ambiguity, cable correction, etc.)
2. Referred to the quasi-absolute gyroscope rather than International Terrestrial Reference Frame
3. Free of ITRF troubles (telescope deformation, co- and post-seismic displacement, non-tidal loading, etc.)

$$\Omega = 7.292 \cdot 10^{-5}$$

Correlation

$$\phi(\omega,t) = \phi_0(\omega_0,t_0) + \frac{\partial\phi}{\partial\omega}(\omega-\omega_0) + \frac{\partial\phi}{\partial t}(t-t_0)\ , \qquad + \frac{\partial^2\phi}{\partial t^2}(t-t_0)^2$$

$$\tau = \frac{r_2 - r_1}{c} \sim 10^{-2} sec \qquad \frac{w_2 - w_1}{c} \sim 10^{-6} \qquad \frac{a_2 - a_1}{c} \sim 10^{-10}\frac{1}{sec}$$

$$\sigma_\tau \sim 10^{-12} sec \qquad \sigma_{\Delta f} \sim 10^{-14} \qquad \sigma_a \sim 10^{-16}\frac{1}{sec}$$

Group delay — **Delay rate** — **Delay rate rate**

Very traditional model is used at the post-correlation analysis

Can we extend the number of observable values to get a new product, kinematic and gravitational acceleration of VLBI sites?

$$\Omega = 7.292 \cdot 10^{-5}$$

Conclusion

1. VLBI delay rate (frequency shift) could be extremely useful for scientific application (Earth rotation, independent catalogue of positions of the reference radio sources)
2. Instantaneous Earth rotation vector could be estimated *(easily)*
3. Is it possible to extract more information from the standard set of VLBI data by extending the model for post-correlation analysis?
4. A link to the Earth gravitational field could be probed.

$$\Omega = 7.292 \cdot 10^{-5}$$

Australian Government
Geoscience Australia

Thank you for your attention

Any Questions?

TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

Astronomical Institute of the Czech Academy of Sciences